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IEEE Standard for Interval Arithmetic (Simplified)

IEEE Computer Society

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Microprocessor Standards Committee

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USA

IEEE Std 1788.1™-2017

IEEE Standard for Interval Arithmetic (Simplified)

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of the
IEEE Computer Society

Approved 6 December 2017

IEEE-SA Standards Board

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Compared to IEEE Std 1788-2015, this standard aims to be minimalistic, yet to cover much of the functionality needed for interval computations. As such, it is more accessible and will be much easier to implement, and thus will speed up production of implementations.

Keywords: arithmetic, computing, decoration, enclosure, interval, IEEE 754™, IEEE 1788™, operation, verified.

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PDF: ISBN 978-1-5044-4602-0 STD22946
Print: ISBN 978-1-5044-4603-7 STDPD22946

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Introduction

This introduction is not part of IEEE Std 1788.1™-2017, IEEE Standard for Interval Arithmetic (Simplified).

This standard is a simplified version of the (full) IEEE Std 1788-2015 for Interval Arithmetic. As such, IEEE Std 1788.1-2017 features

a) set-based intervals,

b) the inf-sup interval type based on the IEEE Std 754-2008 binary64 floating-point format,

c) the decoration system, and

d) simplified I/O.

The full standard extends this standard with extra features, aimed at current applications of interval computation and at anticipated future developments, such as the following.

– Finite-precision intervals of different kinds are supported, e.g. one can build a system where interval endpoints are numbers from an arbitrary-precision floating-point system such as MPFR.

– Besides the model in IEEE Std 1788.1-2017, where a mathematical interval is any closed connected subset of the real numbers, other foundational models are supported in a controlled way by the concept of *flavors*. For instance a flavor may support half-open and open real intervals; or Kaucher/modal intervals like [4,3] as well as [3,4].

  IEEE Std 1788.1-2017 satisfies all General Requirements of IEEE Std 1788-2015 and can be considered a flavor of that standard.

– The set of required operations is extended by operations useful in specialized applications, such as reverse-mode functions for constraint programming, and two-output division used in the interval Newton root-finding method.

This document consists of two parts. Part 1 contains general requirements: overview, scope and purpose of this standard and various definitions and abbreviations. Part 2 is the actual standard, presented in four levels. Level 1 summarizes the theory of set-based intervals including decorations; Level 2 is about representing Level 1 entities in finite precision and the corresponding operations; Level 3 represents Level 2 entities, and Level 4 specifies interchange encodings. Annex A lists the features of IEEE Std 1788-2015 that are not required in IEEE Std 1788.1-2017.

Contents

IEEE Standard for Interval Arithmetic (Simplified)

PART 1

General Requirements

1. Overview

1.1 Scope

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1.2 Purpose

Compared to IEEE Std 1788-2015, this standard aims to be minimalistic, yet to cover much of the functionality needed for interval computations. As such, it is more accessible and will be much easier to implement, and thus will speed up production of implementations.

1.3 Word usage

In this document three words are used to differentiate between different levels of requirements and optionality, as follows:

– **may** indicates a course of action permissible within the limits of the standard with no implied preference ("may" means "is permitted to");

– **shall** indicates mandatory requirements strictly to be followed to conform to the standard and from which no deviation is permitted ("shall" means "is required to");

– **should** indicates that among several possibilities, one is recommended as particularly suitable, without mentioning or excluding others; or that a certain course of action is preferred but not necessarily required; or that (in the negative form) a certain course of action is deprecated but not prohibited ("should" means "is recommended to").

Further:

– **optional** indicates features that may be omitted, but if provided shall be provided exactly as specified;

– **can** is used for statements of possibility and capability ("can" means "is able to");

– **might** indicates the possibility of a situation that could occur, with no implication of the likelihood of that situation ("might" means "could possibly");