



IEEE Standard Specification for Public Key Cryptographic Techniques Based on Hard Problems over Lattices

IEEE Computer Society

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IEEE Standard Specification for Public Key Cryptographic Techniques Based on Hard Problems over Lattices

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IEEE Computer Society

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Abstract: Specifications of common public key cryptographic techniques based on hard problems over lattices supplemental to those considered in IEEE Std 1363-2000 and IEEE Std 1363a-2004, including mathematical primitives for secret value (key) derivation, public key encryption, identification and digital signatures, and cryptographic schemes based on those primitives are provided. Also presented are specifications of related cryptographic parameters, public keys, and private keys. Class of computer and communications systems is not restricted.

Keywords: encryption, lattice-based cryptography, public key cryptography

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Introduction

This introduction is not part of IEEE Std 1363.1-2008, IEEE Standard Specification for Public Key Cryptographic Techniques Based on Hard Problems over Lattices.

The IEEE P1363™ project started as the “Standard for Rivest-Shamir-Adleman, Diffie-Hellman, and Related Public Key Cryptography” with its first meeting in January 1994, following a strategic initiative by the Microprocessor Standards Committee to develop standards for cryptography. Over the next eight years, the working group produced a broad standard reflecting the state of the art in public key cryptography, including techniques from three major families of hard problems. In addition, the working group drafted an addendum that provides additional techniques from those three major families. A more thorough history of the IEEE P1363 working group and its contributions beyond IEEE Std 1363™-2000 are given in the Introduction to IEEE Std 1363-2000.

At the same time, new cryptographic research was producing additional families of cryptographic techniques such as the family of techniques based on hard problems over lattices. These techniques enjoy operating characteristics that make them attractive in certain implementation environments, and thus they have received considerable scrutiny and deployment.

As a result, the working group proposed a new project to standardize techniques from this family. This project was approved by the Microprocessors and Microcomputers Standards Committee, and this current standard is the result of this project.

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1. Overview

1.1 Scope

This standard provides specifications of common public key cryptographic techniques based on hard problems over lattices supplemental to those considered in IEEE Std 1363TM-2000 [B47]¹ and IEEE Std 1363aTM-2004 [B48], including mathematical primitives for secret value (key) derivation, public key encryption, identification and digital signatures, and cryptographic schemes based on those primitives. Specifications of related cryptographic parameters, public keys, and private keys are also presented. Class of computer and communications systems is not restricted.

1.2 Purpose

The transition from paper to electronic media brings with it the need for electronic privacy and authenticity. Public key cryptography offers fundamental technology addressing this need. Many alternative public key techniques have been proposed, each with its own benefits. IEEE Std 1363-2000 [B47] and IEEE Std 1363a-2004 [B48] have produced a comprehensive reference defining a range of common public key

¹ The numbers in brackets correspond to those of the bibliography in Annex B.